10.2 Videos Guide

10.2a

- Calculus with curves defined by parametric equations $x = f(t), \quad y = g(t)$
  - Derivatives
    - $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}, \qquad$ if $\frac{dx}{dt} \neq 0$
    - $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{g'(t)}{f'(t)}\right)}{\frac{dx}{dt}}$
  - Area
    - $A = \int_a^b y\, dx = \int_\alpha^\beta g(t) f'(t)\, dt$
  - Arc length
    - $L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_\alpha^\beta \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$
  - Area of a surface of revolution
    - By rotating about the x-axis: $S = \int_\alpha^\beta 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
    - By rotating about the y-axis: $S = \int_\alpha^\beta 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Exercises:

10.2b

- Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.
  $x = e^t \sin \pi t, \qquad y = e^{2t}, \qquad t = 0$

10.2c

- Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?
  $x = t^2 + 1, \quad y = e^t + 1$

10.2d

- Find points on the curve where the tangent is horizontal or vertical.
  $x = t^3 - 3t, \quad y = t^3 - 3t^2$

10.2e

- Find the area of the region enclosed by the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$. Use a graphing device to graph the curve for $0 \le \theta \le 2\pi$ and an a-value of your choosing.

10.2f

- Find the exact length of the curve.
  $x = 3\cos t - \cos 3t, \qquad y = 3\sin t - \sin 3t, \qquad 0 \le t \le \pi$

10.2g

- Find the exact area of the surface obtained by rotating the given curve about the x-axis.
  $x = 2t^2 + \frac{1}{t}, \qquad y = 8\sqrt{t}, \qquad 1 \le t \le 3$